SOLUTION TO WORKED EXAMPLE: TIMBER TRUSS

structurescentre@gmail.com

The figure shown below is a section of the mono-pitch roof over the gallery of large church auditorium. In other to provide a large uninterrupted space, the use of timber trusses is being considered as an alternative to the use of steel sections. Produce sufficient calculation to establish the forms and sizes of the timber elements using C24 timber.

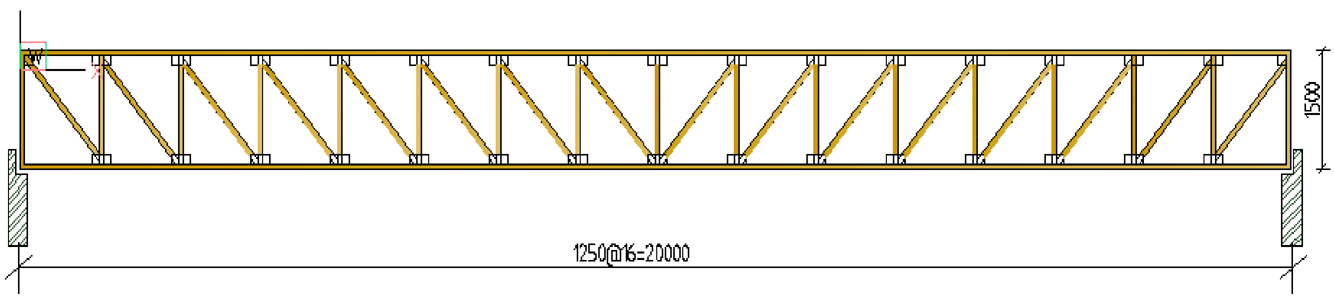

Figure 1: Roof Truss Section

Top Chords

The critical force occurs on member 15-17. Hence the design values will be used to size the top chords.

Max Tensile Force, $\mathbf{N_{Ed,t} = 54.9kN}$

Max Compressive Force, $\mathbf{N_{Ed,c} = 159.0kN}$

Try 150 x 150 Timber Section

Geometric Properties

L = 1.25m

Effective width, L_e= 1.0×l = 1.0×1250 = 1250mm

b = 150mm; h = 150mm;

A = 150 × 150 =22500mm^2

$$I = \frac{bh^3}{12} = \frac{150 \times 150^3}{12} = 42.2 \times 10^6 mm^4$$

$$i = i_{yy} = i_{zz} = \sqrt{\frac{I}{A}} = \sqrt{\frac{42.2 \times 10^6}{22500}} = 43.3mm$$

Tensile resistance

Design tensile stress

$$\sigma_{t,0,d} = \frac{N_{Ed,t}}{A} = \frac{54.9 \times 10^3}{22500} = 2.4N/mm^2$$

Design tensile resistance

$$f_{t,0,d} = \frac{k_h\, k_{sys}\, k_{mod}}{\gamma_M} f_{t,0,k} = \frac{1.0 \times 1.0 \times 0.7}{1.3} \times 8.5 = 4.6N/mm^2$$

$$\sigma_{t,0,d}(2.4N/mm^2) \leq f_{t,0,d}(4.6NN/mm^2)\ \boldsymbol{o.k}$$

Compressive resistance

Design compressive stress

$$\sigma_{t,0,d} = \frac{N_{Ed,c}}{A} = \frac{159 \times 10^3}{22500} = 7.1N/mm^2$$

Design compressive resistance

$$slenderness\ ratio\ \lambda = \lambda_y = \lambda_z = \frac{L_e}{i} = \frac{1250}{43.3} = 28.87$$

$$\lambda_{rel,y} = \lambda_{rel,y} = \lambda_{rel,z} = \frac{\lambda}{\pi}\sqrt{\frac{f_{c,0,k}}{E_{0.05}}} = \frac{28.87}{3.142}\sqrt{\frac{21}{7.4 \times 10^3}} = 0.49$$

$$> 1\ (Platic\ behaviour\ applies)$$

$$k_c = k_{cy} = k_{cz} = 0.5[1 + \beta_v(\lambda_{rel} - 0.3) + \lambda_{rel}^2] = 0.5[1 + 0.2(0.49 - 0.3) + 0.49^2] = 0.64$$

$$k_c = k_{c,y} = k_{c,z} = \frac{1}{k_c + \sqrt{k_c + \lambda_{rel}^2}} = \frac{1}{0.64 + \sqrt{0.64^2 + 0.49^2}} = 0.69$$

$$f_{c,0,d} = \frac{k_{sys}k_c k_{mod}}{\gamma_M} f_{c,0,k} = \frac{1.0 \times 0.69 \times 0.7}{1.3} \times 21 = 7.8N/mm^2$$

$$\sigma_{c,0,d}(7.1N//mm^2) \leq f_{c,0,d}(7.8N/mm^2)\ \boldsymbol{o.k}$$

Adopt the 150 x 150 C24 timber for Top chords.

Bottom Chords

The critical force occurs on member 16-17. The bottom chords are less critical compared to the top chords. However, for simplicity and to ease fabrication, same section as the top chord will be utilized.

Max Tensile Force, $\mathbf{N_{Ed,t}}$ **=156.7kN**

Max Compressive Force, $\mathbf{N_{Ed,\ c}}$ **=54.1kN**

Try 200 x 200 Timber Section

Geometric Properties

L = 1.25m

Effective width, L_e= 1.0×l = 1.0×1250 = 1250mm

b = 200mm; h = 200mm;

A = 200 × 200 =40000mm^2

$$I = \frac{bh^3}{12} = \frac{200 \times 200^3}{12} = 133.3 \times 10^6 mm^4$$

$$i = i_{yy} = i_{zz} = \sqrt{\frac{I}{A}} = \sqrt{\frac{133.3 \times 10^6}{40000}} = 57.7mm$$

Tensile resistance

Design tensile stress

$$\sigma_{t,0,d} = \frac{N_{Ed,t}}{A} = \frac{156.7 \times 10^3}{40000} = 3.9N/mm^2$$

Design tensile resistance

$$f_{t,0,d} = \frac{k_h\, k_{sys}\, k_{mod}}{\gamma_M} f_{t,0,k} = \frac{1.0 \times 1.0 \times 0.7}{1.3} \times 8.5 = 4.6N/mm^2$$

$$\sigma_{t,0,d}(3.9N/mm^2) \leq f_{t,0,d}(4.6NN/mm^2)\ \boldsymbol{o.k}$$

Compressive resistance

Design compressive stress

$$\sigma_{t,0,d} = \frac{N_{Ed,c}}{A} = \frac{54.1 \times 10^3}{40000} = 1.4N/mm^2$$

Design compressive resistance

$$slenderness\ ratio\ \lambda = \lambda_y = \lambda_z = \frac{L_e}{i} = \frac{1250}{57.7} = 21.7$$

$$\lambda_{rel,y} = \lambda_{rel,y} = \lambda_{rel,z} = \frac{\lambda}{\pi}\sqrt{\frac{f_{c,0,k}}{E_{0.05}}} = \frac{21.7}{3.142}\sqrt{\frac{21}{7.4 \times 10^3}} = 0.37$$

$$> 1\ (Platic\ behaviour\ applies)$$

$$k_c = k_{cy} = k_{cz} = 0.5[1 + \beta_v(\lambda_{rel} - 0.3) + \lambda_{rel}^2] = 0.5[1 + 0.2(0.37 - 0.3) + 0.37^2] = 0.58$$

$$k_c = k_{c,y} = k_{c,z} = \frac{1}{k_c + \sqrt{k_c + \lambda_{rel}^2}} = \frac{1}{0.58 + \sqrt{0.58^2 + 0.37^2}} = 0.79$$

$$f_{c,0,d} = \frac{k_{sys} k_c k_{mod}}{\gamma_M} f_{c,0,k} = \frac{1.0 \times 0.79 \times 0.7}{1.3} \times 21 = 8.9 N/mm^2$$

$$\sigma_{c,0,d}(1.4N//mm^2) \leq f_{c,0,d}(8.9N/mm^2)\ \boldsymbol{o.k}$$

Adopt the 200 x 200 C24 timber for Bottom chords.

Diagonals & Verticals

The critical force occurs on members 1-4 and 1-2 for the diagonals and verticals respectively.

Max Tensile Force, $\mathbf{N_{Ed,t}}$ **=58.3kN**

Max Compressive Force, $\mathbf{N_{Ed,\,c}}$ **=47.7kN**

Try 125 x 125 Timber Section

Geometric Properties

L = 1.95m (critical)

Effective width, L_e= 1.0×l = 1.0×1950 = 1950mm

b = 125mm; h = 125mm;

A = 125 × 125 =15625mm^2

$$I = \frac{bh^3}{12} = \frac{125 \times 125^3}{12} = 20.3 \times 10^6 mm^4$$

$$i = i_{yy} = i_{zz} = \sqrt{\frac{I}{A}} = \sqrt{\frac{20.3 \times 10^6}{15625}} = 36.1mm$$

Tensile resistance

Design tensile stress

$$\sigma_{t,0,d} = \frac{N_{Ed,t}}{A} = \frac{58.3 \times 10^3}{15625} = 3.7N/mm^2$$

Design tensile resistance

$$f_{t,0,d} = \frac{k_h\ k_{sys}\ k_{mod}}{\gamma_M} f_{t,0,k} = \frac{1.04 \times 1.0 \times 0.7}{1.3} \times 8.5 = 4.8N/mm^2$$

$$\sigma_{t,0,d}(3.7N/mm^2) \leq f_{t,0,d}(4.8NN/mm^2)\ \boldsymbol{o.k}$$

<u>Compressive resistance</u>

Design compressive stress

$$\sigma_{t,0,d} = \frac{N_{Ed,c}}{A} = \frac{47.7 \times 10^3}{15625} = 3.1N/mm^2$$

Design compressive resistance

$$slenderness\ ratio\ \lambda = \lambda_y = \lambda_z = \frac{L_e}{i} = \frac{1950}{36.1} = 54.0$$

$$\lambda_{rel,y} = \lambda_{rel,y} = \lambda_{rel,z} = \frac{\lambda}{\pi}\sqrt{\frac{f_{c,0,k}}{E_{0.05}}} = \frac{54.0}{3.142}\sqrt{\frac{21}{7.4 \times 10^3}} = 0.92$$

$$> 1\ (Platic\ behaviour\ applies)$$

$$k_c = k_{cy} = k_{cz} = 0.5[1 + \beta_v(\lambda_{rel} - 0.3) + \lambda_{rel}^2] = 0.5[1 + 0.2(0.92 - 0.3) + 0.92^2] = 0.99$$

$$k_c = k_{c,y} = k_{c,z} = \frac{1}{k_c + \sqrt{k_c + \lambda_{rel}^2}} = \frac{1}{0.99 + \sqrt{0.99^2 + 0.92^2}} = 0.43$$

$$f_{c,0,d} = \frac{k_{sys}k_c k_{mod}}{\gamma_M} f_{c,0,k} = \frac{1.0 \times 0.43 \times 0.7}{1.3} \times 21 = 4.9N/mm^2$$

$$\sigma_{c,0,d}(3.1N//mm^2) \leq f_{c,0,d}(4.9N/mm^2)\ \boldsymbol{o.k}$$

<u>Adopt the 125 x 125 C24 timber for Diagonals & Verticals.</u>

Serviceability – Check for Deflection

The deflection of the timber truss can be verified using similar procedure employed in the design of the steel option. In this case, a timber truss, the permissible deflection according to EC5 is limited to span/150.

To determine the maximum deflection occurring at midspan, the truss is idealized as a simply supported beam subjected to a concentrated force. The maximum deflection occurring at midspan is then determined using principle of super-position.

$$\Delta_{max} = \sum \frac{Pa(l-x)(2lx - x^2 - a^2)}{6EIl}$$

Where:

- Δ_{max}= deflection (mm)
- P = Concenterated force at x (N)
- x = point at which deflection is to be found (mm)
- a = point of application of point load from support (mm)
- l= span (mm)
- E = Elastic modulus of members (N/mm^2)
- I = Second moment area of the truss (mm^4)

In the case of the timber truss the permissible deflection according to EC5 is limited to span/150.

Design Summary

- Top Chord = 150 x 150 C24
- Bottom Chord = 200 x 200 C24
- Diagonals & Verticals = 125 x 125 C24